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Research Article

A Full-fuzzy Economic Order Quantity Model with Deterioration, Inflation, and Shortage using ϵ-spread Method

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KEYWORDS

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signed distance defuzzification

ABSTRACT

This paper develops a comprehensive fuzzy economic order quantity (EOQ) model that incorporates deterioration, inflation, and shortage under a full-fuzzy decision-making environment. Unlike classical EOQ formulations that rely on precise parameter values, the proposed approach represents all major cost and system parameters—including demand, ordering cost, holding cost, shortage cost, deterioration rate, and inflation rate—as triangular fuzzy numbers. The fuzzified model is constructed using fuzzy arithmetic operations, and the resulting fuzzy total cost is converted into a crisp objective function through the signed-distance defuzzification method. The defuzzified cost function remains analytically tractable, strictly convex, and guarantees a unique optimal order quantity. A detailed numerical example is provided to illustrate the development of the fuzzy total cost structure, the derivation of the optimality conditions, and the computation of the optimal solution. Sensitivity analysis is conducted by varying each input parameter within a ±20% range, demonstrating the robustness of the model and highlighting the relative influence of system parameters on the optimal inventory decision. The results reveal that deterioration and inflation significantly affect replenishment policies, and the fuzzy modelling framework offers improved flexibility and realism for inventory systems operating under uncertain and fluctuating economic conditions.
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1. Introduction

Inventory systems play a central role in coordinating material flows, stabilising supply chain performance, and supporting production–distribution activities in modern industrial environments. The decision of how much and when to order is influenced by several operational factors such as demand patterns, deterioration of stored goods, replenishment costs, inflationary pressures, and the likelihood of shortages. In recent years, increased uncertainty in supply chains and the growing complexity of market dynamics have made traditional crisp inventory models inadequate for realistic decision-making. Uncertainty arising from vagueness, imprecision, human judgment, and incomplete information cannot always be captured through deterministic or probabilistic approaches. Consequently, many researchers have adopted fuzzy set theory for modelling inventory-related ambiguity, following the seminal concept of fuzzy sets introduced by Zadeh [1]. Fuzzy modelling enables the use of linguistic characterisations and membership functions, thereby offering flexibility in representing ambiguous cost structures and imprecise system parameters.

In real operational settings, one of the major challenges in inventory planning is the deterioration of stored items. Products such as pharmaceuticals, chemicals, agricultural goods, packaged foods, and electronic components lose value or usability over time due to biological decay, moisture, spoilage, damage, or obsolescence. Several recent studies demonstrate that deterioration significantly alters replenishment decisions and cost structures. For example, Behera [2] developed a fuzzy inventory model for perishable items influenced by carbon sensitivity, while Rani et al. [3] examined deteriorating items in green supply chains using fuzzy characterisations. Similarly, Nayak et al. [4] formulated models for Weibull-type deteriorating products under fully backlogged shortages. These studies confirm that deterioration is a crucial factor requiring careful analytical treatment, especially when combined with other uncertain variables.

Another important element affecting inventory costs is inflation. Economic environments characterised by rising prices influence ordering, holding, purchasing, and shortage costs over time. Inflationary conditions are particularly relevant in long planning horizons.Recent works such as Barman et al.[5], who studied inflation in a cloudy fuzzy setting, and Alamri [6], who incorporated preservation technology and carbon emissions under fuzzy inflation, highlight the importance of simultaneously modelling inflation and fuzziness. Inventory decisions that ignore inflation may underestimate true total costs, leading to suboptimal procurement.

Shortages constitute yet another dimension of practical inventory systems. Customer backlogging, lost sales, and delayed fulfillment introduce significant economic consequences. To address this, researchers have analysed shortages in various fuzzy environments. Arora et al.[7] proposed a fuzzy EOQ model with credibility-based demand and shortages. De and Ojha [8] developed a backlogging EOQ model using the structure of a fuzzy Hasse diagram. Poswal et al. [9] studied the effect of price-sensitive demand with shortages in fuzzy settings, while Padiyar et al. [10] analysed deteriorating items with ramp-type demand under cloudy fuzzy environments. These contributions emphasise the need to incorporate shortages into realistic EOQ systems under fuzzy conditions.

Although prior research has investigated deterioration, inflation, and shortages independently or in partial combinations, there remains a lack of comprehensive inventory models that integrate all three factors under a full-fuzzy framework. Recent developments, including the works of Kalaichelvan et al.[11] on fuzzy-machine learning EOQ optimisation and Maity et al. [12] on intuitionistic fuzzy backorder models, illustrate in- creasing interest in sophisticated fuzzy modelling tools. However, a systematic and fully fuzzy EOQ model where all parameters—demand, ordering cost, holding cost, short- age cost, deterioration rate, and inflation rate—are represented as fuzzy numbers is still largely unexplored. This gap motivates the present work.

1.1. Literature Review

The research contributions relevant to fuzzy EOQ models with deterioration, inflation, and shortages can be classified into four major categories: (i) fuzzy EOQ models and generalised fuzzy inventory systems, (ii) deterioration-based fuzzy inventory models, (iii) inflationary fuzzy EOQ models, and (iv) fuzzy EOQ with shortages and backlogging. The following sections synthesize the literature, incorporating all references provided.

Fuzzy EOQ and generalised fuzzy inventory models. Early foundations of fuzzy EOQ modelling were established by Park [13], after which several researchers extended the framework using advanced fuzzy representations. Arora et al. [7] analysed credibility-based fuzzy demand and shortages. Maity et al. [12] introduced intuitionistic fuzzy demand into a backorder EOQ system. Kalaichelvan et al. [11] integrated fuzzy theory with machine learning to optimise EOQ in pharmaceutical settings, while Maity et al. [14]studied pentagonal intuitionistic fuzzy systems with carbon emission effects. A number of studies focus on fuzzy learning mechanisms and fuzzy parameter evolution. Ganesan and Uthayakumar contributed significantly to this line of research. In [15], they incorporated a learning function into fuzzy parameters in an EOQ model with backorders, while in [16] they proposed fuzzy demand in a two-echelon supply chain model with learning effects. Furthermore, Ganesan and Uthayakumar [17] developed a sustainable fuzzy EPQ model requiring the solution of a sextic equation, highlighting the role of higher-order fuzzy structures in inventory optimisation. Additional studies have expanded the domain of fuzzy inventory systems. Malik and Garg [18] examined fuzzy two-warehouse models with complex storage and deterioration behaviour. Kumar et al. [19] analysed seasonal fuzzy demand for deteriorating items, extending fuzzy EOQ theory to cyclical market environments. Jayaswal et al. [20] explored imperfect-quality items under cloudy fuzzy conditions with trade credit. These works collectively show a growing integration of fuzzy modelling with learning, seasonal dynamics, and multi-storage mechanisms.

Fuzzy inventory models for deteriorating items. Deterioration is a core feature in many inventory systems. Several studies have incorporated fuzzy deterioration rates to model uncertainty more accurately. Behera [2] developed a fuzzy model for perishable products with carbon sensitivity. Rani et al.[3] formulated a fuzzy deteriorating inventory model involving refurbished items and cannibalisation processes. Senbagam and Kokilamani proposed Gompertz deterioration models in fuzzy environments with linear [21] and cubic demand [22]. Nayak et al. [4] considered Weibull deterioration under fully backlogged shortages. Other important contributions include Das [23], who proposed a fuzzy multi-objective model with demand -dependent deterioration and fuzzy lead times. Moreover, Kumar et al. [24] developed two-storage fuzzy models incorporating time-dependent deterioration, and Kumar et al. [25] analysed fuzzy demand with deterioration in a cost-optimisation setting. Manna et al. [26] extended deterioration modelling to imperfect production systems under fuzzy assumptions. Together, these studies provide strong evidence that deterioration must be paired with uncertainty, although integration with fuzzy inflation and shortages remains limited.

Fuzzy inflationary models in inventory research. Inflation affects replenishment cycles by modifying the real value of ordering, holding, and shortage costs. Barman et al. [5] developed a cloudy fuzzy inflation-based model for backorders. Alamri [6] integrated inflation with carbon emissions, trade credit, and preservation technology under fuzzy parameters. Kumar et al. [27] proposed a sustainable fuzzy inventory model for deteriorating items with inflation and partial backlogging. Jayaswal et al. [20] also addressed inflation effects in a cloudy fuzzy setting. Moreover, the influence of learning and cost evolution in fuzzy systems was studied by Ganesan and Uthayakumar [16], who demonstrated that learning interacts significantly with cost variability. These contributions reflect the increasing academic interest in capturing inflation within fuzzy EOQ systems. Still, most studies treat only selected parameters as fuzzy and do not combine inflation simultaneously with deterioration and shortages.

Fuzzy EOQ models with shortages and backlogging. Shortages, whether partially or fully backlogged, are common in practical inventory systems. Several works incorporate shortages into fuzzy EOQ models. Arora et al. [7] used credibility-based fuzzy demand in shortage situations. Poswal et al. [9] examined fuzzy EOQ with price-sensitive demand and shortages. Padiyar et al. [10] extended shortage modelling to supply-chain settings under cloudy fuzzy conditions. De and Ojha [8] introduced a novel use of fuzzy Hasse diagrams to handle backlogging behaviour, demonstrating the potential of algebraic fuzzy structures in inventory research. Additional contributions include models with learning and fuzzy shortages proposed by Ganesan and Uthayakumar [15], as well as seasonal demand shortages investigated by Kumar et al.[19]. These studies collectively indicate that shortages are well researched in fuzzy environments; however, they do not fully integrate shortages with inflation and deterioration in a unified fuzzy framework.

1.2. Research Gap

Despite substantial advancements in fuzzy inventory modelling, several important limitations remain:

- Existing fuzzy EOQ models often fuzzify only a subset of parameters such as demand or holding cost, whereas a comprehensive treatment of all major parameters (demand, ordering cost, holding cost, shortage cost, deterioration rate, and inflation rate) in a unified framework is largely absent.
- Deterioration, shortages, and inflation have been individually studied in fuzzy environments, but very few models integrate all three components simultaneously, and none of the reviewed studies report a full-fuzzy EOQ formulation using the ϵ-spread method.
- Approaches such as intuitionistic fuzzy sets, cloudy fuzzy numbers, and pentagonal fuzzy representations have been used, yet systematic fuzzification using ϵ-spread remains largely unexplored for EOQ modelling.
- Many prior models lack a mathematically explicit defuzzified cost function that leads to a solvable optimality condition such as a cubic equation, limiting analytical insight.
- Few studies illustrate the combined operational impact of deterioration, inflation, and shortages when every parameter is fuzzy, particularly in terms of how fuzziness propagates through total cost formulation.

These limitations indicate the need for a generalised, mathematically consistent full- fuzzy EOQ model that captures all relevant sources of uncertainty.

1.3. Research Objectives and Contributions

To address the above research gaps, the present study proposes a comprehensive model with the following objectives:

- To formulate a full-fuzzy EOQ model in which all key parameters—demand, ordering cost, holding cost, shortage cost, deterioration rate, and inflation rate—are represented using triangular fuzzy numbers.
- To integrate deterioration, shortages, and inflation into a unified fuzzified structure suitable for real-world inventory systems marked by uncertainty.

- To systematically generate fuzzy parameters using the ϵ-spread method, ensuring internal consistency of fuzzy arithmetic operations.
- To derive a defuzzified total cost function using the signed-distance method, producing a tractable expression suitable for analytic optimisation.
- To obtain the optimal order quantity by solving the cubic optimality condition derived from the first-order derivative of the defuzzified cost function.
- To demonstrate the applicability of the proposed model using a detailed numerical example and analyse the behaviour of optimal solutions.

These contributions position the proposed model as a rigorous and comprehensive framework that broaden the theoretical foundation of fuzzy EOQ modelling while offering practical decision-making tools for environments characterised by multi-dimensional uncertainty.

2. Fundamentals of Fuzzy Quantities

Many real-world decision parameters cannot be described precisely due to incomplete information, qualitative judgments, or environmental variability. Fuzzy set theory offers a powerful tool to model such vagueness by assigning gradual degrees of membership rather than sharp numerical boundaries. Since several parameters in our inventory model—such as demand, deterioration, inflation, and cost components—exhibit inherent imprecision, fuzzy numbers are employed to represent these uncertainties. This section introduces the essential concepts of fuzzy sets, triangular fuzzy numbers, arithmetic operations on fuzzy quantities, and the defuzzification procedure used to extract crisp values from fuzzy representations.

Fuzzy sets and membership representation. Let Z be a universal set whose elements are denoted by z. A fuzzy set A on Z is characterized by its membership function $\mu_A : Z \to [0, 1]$ and is expressed as $A = \{(z, \mu_A(z)) : z \in Z\}$. The value $\mu_A(z)$ indicates the extent to which z belongs to the fuzzy set. The supremum of the membership values is called the height of the fuzzy set. If $\sup_{z \in Z} \mu_A(z) = 1$, the fuzzy set is said to be normalized. When $Z = \mathbb{R}$ and $\mu_A(\lambda z_1 + (1-\lambda) z_2) \geq \min\{\mu_A(z_1), \mu_A(z_2)\}$, for all $\lambda \in [0, 1]$, the fuzzy set is convex. A fuzzy set that is both normalized and convex, with a membership function that is piecewise continuous, is referred to as a fuzzy number.

Triangular fuzzy numbers. A widely used fuzzy number in engineering and decision sciences is the triangular fuzzy number (TFN). Let three real numbers satisfy $l < m < u$. A triangular fuzzy number T is denoted by $T = (l, m, u)$, and has the membership function

$$\mu_T(z) = \begin{cases} \dfrac{z - l}{m - l}, & l \leq z \leq m \\ \dfrac{u - z}{u - m}, & m \leq z \leq u \\ 0, & \text{otherwise} \end{cases}$$

Here, l is the lower bound, m is the most plausible (modal) value, and u is the upper bound. If $m - l = u - m$, the triangular fuzzy number is symmetric; otherwise, it is asymmetric. The intervals $[l, m]$ and $[m, u]$ describe the left and right spreads, indicating how uncertainty is distributed around the modal value.

Arithmetic operations on triangular fuzzy numbers. Let two triangular fuzzy numbers be expressed as $X = (x_L, x_M, x_U)$, $Y = (y_L, y_M, y_U)$ where $x_L < x_M < x_U$ and $y_L < y_M < y_U$. Let h be a strictly positive scalar. The fundamental arithmetic operations are defined component-wise as follows.

- **Addition:** $\mathbb{X} + \mathbb{Y} = (x_L + y_L,\ x_M + y_M,\ x_U + y_U)$
- **Subtraction:** $\mathbb{X} - \mathbb{Y} = (x_L - y_U,\ x_M - y_M,\ x_U - y_L)$
- **Scalar multiplication:** $h\mathbb{X} = (h x_L,\ h x_M,\ h x_U)$

- **Multiplication:** If all components of both fuzzy numbers are strictly positive, $\mathbb{X}\mathbb{Y} = (x_L y_L, x_M y_M, x_U y_U)$.
- **Division:** Under the same positivity assumption, $\mathbb{X}/\mathbb{Y} = \left(\frac{x_L}{y_U}, \frac{x_M}{y_M}, \frac{x_U}{y_L}\right)$.

These operations enable the manipulation of uncertain quantities directly through fuzzy arithmetic, which is essential when constructing and solving fully fuzzy inventory models.

Defuzzification. In optimization problems, it is often necessary to convert a fuzzy quantity into a single crisp value. This process, known as defuzzification, generates a representative real number by aggregating the information contained in the member- ship function. For a triangular fuzzy number *T*= (l, m, u), the signed-distance–based defuzzification method yields

$$\Delta(\mathbb{T}) = \frac{l + 2m + u}{4}$$

This formula assigns twice as much weight to the modal value compared to the two bounds, reflecting the central tendency of the fuzzy number while still providing a balanced measure of uncertainty on either side.

3. Base Inventory Model

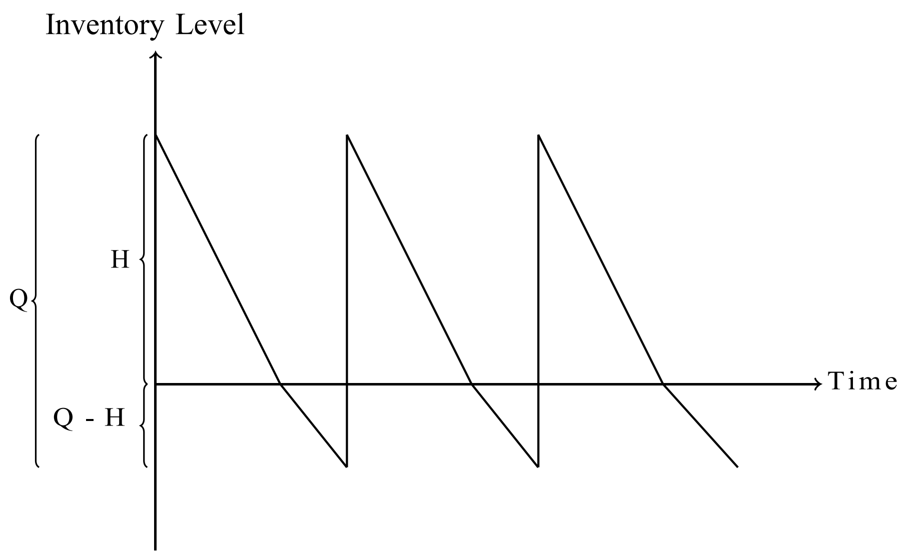

Figure 1. Inventory behaviour with shortages and backlogging.

The Economic Order Quantity (EOQ) framework remains one of the most influential foundations in inventory theory, providing a systematic method for determining replenishment quantities that minimize total annual cost. In many real operational systems, shortages are allowed and fully backlogged, leading to a cyclic behaviour in which each replenishment cycle consists of a positive inventory period followed by a shortage period. When deterioration and inflation are present, the structure of the cost function becomes richer and more realistic, as illustrated in Figure 1. This classical cyclic pattern provides the basis for constructing the cost model before extending it to a fuzzy setting. To formulate the crisp model, we define several parameters related to demand, ordering activities, holding and shortage penalties, deterioration, and inflation. These parameters are listed in Table 1.

Table 1. Notation for the base inventory model.

Symbol	Description
D	Annual demand rate (units/year)
K	Ordering cost per order (currency/order)
h	Holding cost per unit per year (currency/unit/year)
p	Shortage penalty cost per unit (currency/ unit)
δ	Deterioration rate (fraction of loss)
f	Inflation cost parameter (currency/unit)
Q	Order quantity (units)
H	Maximum on-hand inventory (units)

The annual demand is denoted by D, and the system replenishes inventory in batches of size Q. Each replenishment incurs a fixed ordering cost K, and the number of orders per year is D/Q, leading to an annual ordering cost of KD/Q. During each cycle, the inventory rises instantly to its maximum level H and then decreases linearly due to demand consumption. Since the positive inventory profile forms a triangular shape, its average magnitude is $H/2$. Multiplying this by the holding cost h and adjusting for deterioration via $(1 - \delta)$ yields an annual holding cost of $\frac{1}{2}hQ(1 - \delta)$, consistent with widely used formulations in deterioration-based inventory models.

The shortage portion also forms a triangular region, reaching a maximum backlog of $Q - H$. The average shortage level is $(Q - H)/2$, and multiplying this by the shortage penalty cost p leads to a shortage cost expression proportional to the area of the shortage triangle. Following standard EOQ-shortage formulations, the annual shortage penalty becomes

$$p\frac{(D-Q)^2}{2D}$$

Finally, inflation affects cost components through purchasing power or cost escalation effects. Using a linear inflation parameter, the inflation contribution for each replenishment quantity is represented as fQ, analogous to inflation-adjusted EOQ formulations. Combining all components, the crisp total annual cost becomes

$$TC(Q) = K\frac{D}{Q} + \frac{1}{2}hQ(1-\delta) + p\frac{(D-Q)^2}{2D} + fQ \tag{1}$$

Equation (1) represents the base model for our study, incorporating ordering cost, inventory holding with deterioration, shortage penalty, and inflation effects. In the subsequent section, this deterministic framework will be extended to a fully fuzzy formulation in which all parameters become fuzzy quantities, enabling the model to capture uncertainty more realistically, consistent with recent trends in fuzzy inventory studies.

4. Fuzzy Inventory Model

The base EOQ formulation in (1) assumes that all parameters are known precisely. In practice, however, several of these quantities are not available as exact numbers. Annual demand, ordering cost, holding cost, deterioration rate, shortage penalty, and inflation are often estimated from incomplete data, expert opinion, or uncertain forecasts. Consequently, the use of point estimates may misrepresent the underlying uncertainty and lead to sub–optimal or misleading decisions. Fuzzy set theory provides a systematic way to model such imprecision by representing uncertain quantities as fuzzy numbers rather than crisp scalars. In this section, the base model is extended into a fully fuzzy environment in which all key parameters are described by triangular fuzzy numbers. We first introduce the fuzzy notation for each parameter, then derive the fuzzy total cost function using triangular fuzzy arithmetic, and finally apply a signed–distance defuzzification scheme to obtain a usable crisp objective function in terms of the decision variable Q.

4.1. Fuzzification of Model Parameters

Let all major cost and demand parameters be represented by triangular fuzzy numbers. Specifically, we define

$\widehat{D} = (D_L, D_M, D_U)$	Annual demand (fuzzy)
$\widehat{K} = (K_L, K_M, K_U)$	Ordering cost per order (fuzzy)
$\hat{h} = (h_L, h_M, h_U)$	Holding cost per unit per year (fuzzy)
$\hat{p} = (p_L, p_M, p_U)$	Shortage cost per unit (fuzzy)
$\hat{\delta} = (\delta_L, \delta_M, \delta_U)$	Deterioration rate (fuzzy)
$\hat{f} = (f_L, f_M, f_U)$	Inflation cost parameter (fuzzy)

Each triple $(\cdot_L, \cdot_M, \cdot_U)$ denotes a triangular fuzzy number with lower, modal, and upper values, as described in the preliminaries section. The decision variable Q is assumed to remain crisp. This choice is common in fuzzy EOQ literature: parameters reflect environmental uncertainty, while Q is the controllable decision chosen by the manager. In many applications, triangular fuzzy numbers are obtained from expert estimates via an ϵ–spread mechanism: if x_M is the best estimate of a parameter and $\epsilon_1, \epsilon_2 > 0$ represent allowable deviations to the left and right, then the fuzzy parameter is taken as $(x_M - \epsilon_1, x_M, x_m + \epsilon_2)$. This procedure allows us to embed subjective judgments and tolerance ranges directly into the model.

4.2. Fuzzy Total Cost Function

The fuzzy counterpart of the crisp total cost (1) is constructed by replacing each crisp parameter by its fuzzy analogue:

$$\widehat{TC}(Q) = \widehat{K}\frac{\widehat{D}}{Q} + \frac{1}{2}\hat{h}Q\big(1 - \hat{\delta}\big) + \hat{p}\frac{\big(\widehat{D} - Q\big)^2}{2\widehat{D}} + \hat{f}Q \tag{2}$$

Here, the arithmetic operations are interpreted in the sense of triangular fuzzy arithmetic. We now examine each term in detail.

Ordering cost term. The fuzzy annual ordering cost is

$$\widehat{C}_{\text{ord}}(Q) = \widehat{K}\frac{\widehat{D}}{Q}$$

Since $Q > 0$, division by Q is a simple scalar operation. Using the triangular multiplication rule for positive components,

$$\widehat{K}\widehat{D} = (K_L, K_M, K_U)(D_L, D_M, D_U) = (K_L D_L, K_M D_M, K_U D_U)$$

and, therefore

$$\widehat{C}_{\mathrm{ord}}(Q) = \left(\frac{K_L D_L}{Q}, \frac{K_M D_M}{Q}, \frac{K_U D_U}{Q}\right) \tag{3}$$

Holding cost term with deterioration. The fuzzy holding cost term is given by

$$\widehat{C}_{\mathrm{hold}}(Q) = \frac{1}{2}\hat{h}Q\big(1 - \hat{\delta}\big)$$

First, we compute the fuzzy quantity $\big(1 - \hat{\delta}\big)$ (1 - δ). If $\hat{\delta} = (\delta_L, \delta_M, \delta_U)$, then

$$1 - \hat{\delta} = (1 - \delta_U, 1 - \delta_M, 1 - \delta_L)$$

since subtracting a triangular fuzzy number from a crisp scalar reverses the order of the bounds. Next, we multiply $\hat{h}$ and $(1 - \hat{\delta})$:

$$\hat{h}\big(1 - \hat{\delta}\big) = (h_L, h_M, h_U)(1 - \delta_U, 1 - \delta_M, 1 - \delta_L) = (h_L(1 - \delta_U), h_M(1 - \delta_M), h_U(1 - \delta_L))$$

Multiplication by the positive scalar $Q/2$ then yields

$$\widehat{C}_{\mathrm{hold}}(Q) = \left(\frac{Q}{2}h_L(1 - \delta_U), \frac{Q}{2}h_M(1 - \delta_M), \frac{Q}{2}h_U(1 - \delta_L)\right) \tag{4}$$

Shortage cost term. The fuzzy shortage cost term is

$$\widehat{C}_{\mathrm{short}}(Q) = \hat{p}\frac{\big(\widehat{D} - Q\big)^2}{2\widehat{D}}$$

We proceed in steps. First, subtract the crisp quantity Q from the fuzzy demand:

$$\widehat{D} - Q = (D_L - Q, D_M - Q, D_U - Q)$$

Assuming $D_L > Q$ for feasibility, the components remain positive. Squaring a positive triangular fuzzy number leads to

$$\big(\widehat{D} - Q\big)^2 = \big((D_L - Q)^2, (D_M - Q)^2, (D_U - Q)^2\big)$$

Next, multiplying by the fuzzy penalty cost $\hat{p}$ gives

$$\hat{p}\big(\widehat{D} - Q\big)^2 = \big(p_L(D_L - Q)^2, p_M(D_M - Q)^2, p_U(D_U - Q)^2\big)$$

We then divide by $2\widehat{D} = (2D_L, 2D_M, 2D_U)$. For positive triangular fuzzy numbers $X = (x_L, x_M, x_U)$ and $Y = (y_L, y_M, y_U)$, the division

$$\frac{X}{Y} = \left(\frac{x_L}{y_U}, \frac{x_M}{y_M}, \frac{x_U}{y_L}\right)$$

Applying this rule, we obtain

$$\widehat{C}_{\mathrm{short}}(Q) = \left(p_L\frac{(D_L - Q)^2}{2D_U}, p_M\frac{(D_M - Q)^2}{2D_M}, p_U\frac{(D_U - Q)^2}{2D_L}\right) \tag{5}$$

Inflation cost term. The fuzzy inflation term is

$$\widehat{C}_{\text{infl}}(Q) = \hat{f}Q$$

Since Q is a positive scalar, this is simply

$$\widehat{C}_{\text{infl}}(Q) = (f_L Q, f_M Q, f_U Q) \tag{6}$$

Aggregate fuzzy total cost. Collecting the expressions (3)–(6), the total annual cost in fuzzy form can be written as a triangular fuzzy number

$$\widehat{TC}(Q) = (T_L(Q), T_M(Q), T_U(Q))$$

where

$$T_L(Q) = \frac{K_L D_L}{Q} + \frac{Q}{2} h_L(1-\delta_U) + p_L \frac{(D_L - Q)^2}{2D_U} + f_L Q \tag{7}$$

$$T_M(Q) = \frac{K_M D_M}{Q} + \frac{Q}{2} h_M(1-\delta_M) + p_M \frac{(D_M - Q)^2}{2D_M} + f_M Q \tag{8}$$

$$T_U(Q) = \frac{K_U D_U}{Q} + \frac{Q}{2} h_U(1-\delta_L) + p_U \frac{(D_U - Q)^2}{2D_L} + f_U Q \tag{9}$$

Each of these functions has a structure similar to the crisp cost function, but evaluated at the lower, modal, and upper endpoints of the triangular fuzzy parameters. This decomposition is particularly useful for sensitivity analysis, allowing one to study optimistic (T_L), most likely (T_M), and pessimistic (T_U) scenarios.

Defuzzification and objective function. To select an optimal order quantity, it is necessary to map the fuzzy total cost $\widehat{TC}(Q)$ to a single scalar cost value for each Q. We employ the signed–distance–based defuzzification method described earlier. For a triangular fuzzy number (x_L, x_M, x_U), the defuzzified value is

$$\Delta(x_L, x_M, x_U) = \frac{x_L + 2x_M + x_U}{4}$$

Applying this to $\widehat{TC}(Q)$, we obtain the defuzzified total cost function

$$\widetilde{TC}(Q) = \Delta(T_L(Q), T_M(Q), T_U(Q)) = \frac{T_L(Q) + 2T_M(Q) + T_U(Q)}{4} \tag{10}$$

Substituting from (7)–(9) yields

$$\begin{aligned}\widetilde{TC}(Q) = &\frac{1}{4}\left[\frac{K_L D_L}{Q} + \frac{Q}{2} h_L(1-\delta_U) + p_L \frac{(D_L - Q)^2}{2D_U} + f_L Q\right] \\ &+ \frac{1}{2}\left[\frac{K_M D_M}{Q} + \frac{Q}{2} h_M(1-\delta_M) + p_M \frac{(D_M - Q)^2}{2D_M} + f_M Q\right] \\ &+ \frac{1}{4}\left[\frac{K_U D_U}{Q} + \frac{Q}{2} h_U(1-\delta_L) + p_U \frac{(D_U - Q)^2}{2D_L} + f_U Q\right]\end{aligned} \tag{11}$$

Equation (11) is the final objective function derived from the fully fuzzy inventory model.The optimal order quantity Q^* is obtained by differentiating $\widetilde{TC}(Q)$ with respect to Q and solving the resulting nonlinear equation. Although the derivative leads to a higher–degree polynomial in Q, numerical methods can be used efficiently. This approach is consistent with several recent fuzzy EOQ investigations, where defuzzified cost functions are minimized numerically to obtain robust inventory decisions under uncertainty.

The fuzzy model thus constructed generalizes the classical EOQ model by allowing all key parameters to be uncertain within specified triangular bounds while preserving a clear optimization structure. In the next section, a numerical illustration will demonstrate how the proposed fuzzy model behaves in comparison with its crisp counterpart and will highlight the influence of parameter fuzziness on optimal policies.

4.3. Solution Procedure and Optimality Analysis

Each term of $T_L(Q)$, $T_M(Q)$ and $T_U(Q)$ consists of a hyperbolic part (ordering cost), a linear term in Q (holding and inflation), and a quadratic term arising from the shortage component. Such functional structures are common in EOQ models with shortages and deterioration. Since the defuzzified cost $\widetilde{TC}(Q)$ is a convex combination (weighted average) of these three functions, it preserves the same general shape. To see this more clearly, we expand the shortage term in each component. For example,

$$\frac{p_L(D_L-Q)^2}{2D_U}=\frac{p_L}{2D_U}\left(Q^2-2D_LQ+D_L^2\right)=\frac{p_L}{2D_U}Q^2-\frac{p_LD_L}{D_U}Q+\frac{p_LD_L^2}{2D_U}$$

and similarly for the M and U cases. Substituting into (11) and collecting like powers of Q yields

$$\widetilde{TC}(Q)=\frac{A}{Q}+BQ^2+CQ+E \tag{12}$$

where A, B, C, E are positive constants determined by the fuzzy parameters. Specifically,

$$A=\frac{1}{4}(K_LD_L+2K_MD_M+K_UD_U)$$

$$B=\frac{1}{4}\left(\frac{p_L}{2D_U}+\frac{p_M}{D_M}+\frac{p_U}{2D_L}\right)$$

$$C=\frac{1}{4}(h_L(1-\delta_U)+2h_M(1-\delta_M)+h_U(1-\delta_L))-\frac{1}{2}\left(\frac{p_LD_L}{2D_U}+\frac{p_UD_U}{2D_L}+p_M\right)+\frac{1}{4}(f_L+2f_M+f_U)$$

$$E=\frac{1}{4}\left(\frac{p_LD_L^2}{2D_U}+p_MD_M+\frac{p_UD_U^2}{2D_L}\right)$$

Note that E is a constant independent of Q. The constant E does not influence the optimal Q, so it may be disregarded in the optimization. All terms in the expression for A come from the product of ordering cost components and demand components. Ordering cost parameters satisfy $K_L \le K_M \le K_U$, $K_L, K_M, K_U > 0$, and the triangular fuzzy demand components satisfy $D_L \le D_M \le D_U$, $D_L, D_M, D_U > 0$. Since the product of two positive numbers is positive, $K_LD_L>0$, $K_MD_M>0$, $K_UD_U>0$. Hence, $K_LD_L+2K_MD_M+K_UD_U>0$. Multiplication by the positive scalar 1/4 preserves positivity, so $A>0$. The constant B arises entirely from the shortage cost component of the fuzzy model.Shortage cost parameters also satisfy $p_L \le p_M \le p_U$, $p_L, p_M, p_U > 0$, and demand parameters satisfy $D_L, D_M, D_U > 0$ .Thus each fraction inside, $\frac{p_L}{2D_U}, \frac{p_M}{D_M}, \frac{p_U}{2D_L}$, is strictly positive because it is a ratio of positive quantities. Therefore, $\frac{p_L}{2D_U}+\frac{p_M}{D_M}+\frac{p_U}{2D_L}>0$, and multiplication by 1/4 preserves positivity. Hence, $B>0$. Since both $A>0$ and $B>0$, the defuzzified cost function $\widetilde{TC}(Q)=\frac{A}{Q}+BQ^2+CQ+E$ consists of a strictly decreasing term A/Q and a strictly convex increasing term BQ^2. As shown in the subsequent convexity analysis, this structure ensures that the overall function is strictly convex on $Q>0$, and therefore admits a unique global minimizer. This form is particularly convenient for analyzing convexity and deriving the optimal solution, as it combines a decreasing hyperbolic part with increasing polynomial terms.

First-order optimality condition. To obtain the optimal order quantity Q^*, we differentiate the defuzzified cost function with respect to Q:

$$\frac{d\widetilde{TC}(Q)}{dQ} = -\frac{A}{Q^2} + 2BQ + C$$

The critical points are obtained by solving the first-order condition:

$$\frac{d\widetilde{TC}(Q)}{dQ} = 0 \Leftrightarrow -\frac{A}{Q^2} + 2BQ + C = 0 \tag{13}$$

Multiplying both sides of (13) by Q^2 (which is positive for $Q > 0$) yields

$$2BQ^3 + CQ^2 - A = 0 \tag{14}$$

Equation (14) is a cubic equation in Q with real coefficients. Since $B > 0$ and $A > 0$, the term $2BQ^3$ dominates as $Q \to +\infty$, while the constant term $-A$ dominates as $Q \to 0^+$. Consequently, the left-hand side of (14) is negative for sufficiently small Q and positive for sufficiently large Q. By continuity, there exists at least one positive real root of (14). This root corresponds to a stationary point of $\widetilde{TC}(Q)$. Closed-form expressions for cubic polynomials are available; however, in practical inventory applications it is customary to solve such equations using numerical methods. Because the cost function is convex (as shown below), this stationary point is guaranteed to be the unique global minimizer.

Convexity of the objective function. To verify that the stationary point arising from (14) corresponds to a global minimum, we analyze the second derivative of $\widehat{TC}(Q)$. Differentiating the first derivative once more, we obtain:

$$\frac{d^2\widetilde{TC}(Q)}{dQ^2} = \frac{2A}{Q^3} + 2B$$

For all $Q > 0$, $A > 0$ and $B > 0$, so each term on the right-hand side is strictly positive. Hence,

$$\frac{d^2\widetilde{TC}(Q)}{dQ^2} > 0 \quad \text{for all} \quad Q > 0 \tag{15}$$

This implies that $\widetilde{TC}(Q)$ is strictly convex on its domain. A strictly convex function on a convex domain possesses at most one stationary point, and any such stationary point is necessarily the unique global minimizer. Therefore, the positive root Q^* of (14) is the unique optimal order quantity for the fuzzy inventory model. This convexity property parallels that of classical EOQ models with shortages and deterioration, in which the cost function typically consists of a hyperbolic term plus convex polynomial terms. The fuzzy extension preserves this structural convexity because the defuzzified cost is simply a weighted combination of convex functions.

Graphical illustration of the objective function. The qualitative behaviour of $\widetilde{TC}(Q)$ as a function of Q can be visualized in Figure 2. For very small order quantities, the ordering cost term A/Q becomes extremely large, dominating the total cost. As Q increases, this term decreases rapidly, but the quadratic and linear terms $BQ^2 + CQ$ start to grow and eventually dominate. Because of the interplay between these opposing effects, the curve exhibits a unique minimum at $Q=Q^*$.

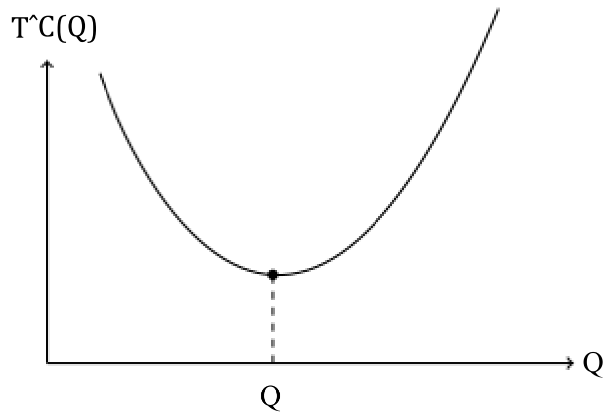

Figure 2. Typical shape of the defuzzified total cost function $\widetilde{TC}(Q)$.

The figure confirms the analytical result: the objective function is convex, decreasing initially due to the reduction in ordering frequency, and then increasing as holding, shortage, and inflation-related costs dominate. The unique minimum reflects the trade- off inherent in EOQ-type decisions. In numerical studies, one can fix parameter values (D_L, D_M, D_U), (K_L, K_M, K_U), etc., evaluate $\widetilde{TC}(Q)$ over a grid of Q values, and visually confirm the convex shape and the position of Q^* , in line with earlier fuzzy EOQ analyses reported in. In summary, the solution procedure for the proposed fuzzy model consists of:

1. Constructing the defuzzified cost function $\widetilde{TC}(Q)$ as in (12).
2. Differentiating to obtain the first-order condition (13) and the cubic equation (14).
3. Solving the cubic equation numerically to find the unique positive root Q^*.
4. Using the convexity result (15) to guarantee that Q^* is the global optimal order quantity.

This provides a complete and rigorous method for determining the optimal policy under the proposed fuzzy EOQ framework.

5. Numerical Example

To illustrate the applicability and computational structure of the proposed fuzzy economic order quantity model with deterioration, inflation, and shortage, this section presents a complete numerical example. The objective is to demonstrate how triangular fuzzy parameters are incorporated into the model, how the fuzzy total cost is constructed, and how the defuzzified cost function is optimized to obtain the economic order quantity. All fuzzy parameters are represented as triangular fuzzy numbers, and defuzzification is carried out using the signed-distance method. The following fuzzy parameters are used in this numerical example, with values expressed in appropriate units:

Parameter	Triangular Fuzzy Number	Units
Demand rate $\widehat{D}$	$(45000, 50000, 55000)$	Kg/year
Ordering cost $\widehat{K}$	$(250, 300, 350)$	INR/order
Holding cost $\hat{h}$	$(3.5, 4.0, 4.5)$	INR/Kg/year
Shortage cost $\hat{p}$	$(0.15, 0.20, 0.25)$	INR/Kg
Deterioration rate $\hat{\delta}$	$(0.10, 0.13, 0.16)$	Proportion/year
Inflation rate $\hat{f}$	$(0.08, 0.10, 0.12)$	Proportion/year

Using fuzzy arithmetic, each cost component in the total cost function is first computed in its lower, middle, and upper triangular forms. These are then combined and defuzzified using $d(a_L, a_M, a_U) = \frac{a_L+2a_M+a_U}{4}$, resulting in a single real-valued total annual cost function. After symbolic simplification, the defuzzified total cost function for this data set is obtained as

$$\widetilde{TC}(Q) = 2.035 \times 10^{-6}Q^2 + 1.637Q + 5291.035 + \frac{15125000}{Q}$$

This cost function is strictly positive, differentiable, and convex for all $Q > 0$. The first derivative is computed as

$$\widetilde{TC}'(Q) = 4.071\times10^{-6}Q + 1.637 - \frac{15125000}{Q^2}$$

Setting $\widetilde{TC}'(Q)=0$ and multiplying by Q^2 gives the cubic equation

$$4.071\times10^{-6}Q^3 + 1.637Q^2 - 15125000 = 0$$

Using *sympy.nroots* of Python, the following roots are obtained:

$$\{-402039.668,\ -3051.549,\ \mathbf{3028.561}\}$$

Only the positive real root is admissible, and hence

$$Q^* = 3028.561 \text{ Kg.}$$

The convexity of the total cost function is confirmed by examining the second derivative:

$$\widetilde{TC}''(Q) = 4.071\times10^{-6} + \frac{30250000}{Q^3}$$

which is strictly positive for all $Q>0$. Therefore, $\widetilde{TC}(Q)$ is strictly convex and possesses a unique global minimizer at $Q=Q^*$. Substituting the optimal value $Q^* = 3028.5612$ into the defuzzified cost function gives $\widetilde{TC}\left(Q^*\right) = 15260.61$ INR per year. This constitutes the minimum annual total cost associated with the chosen fuzzy parameters. Figure 3 displays the total cost curve, along with the vertical indicator at Q^* and the marked minimum point.

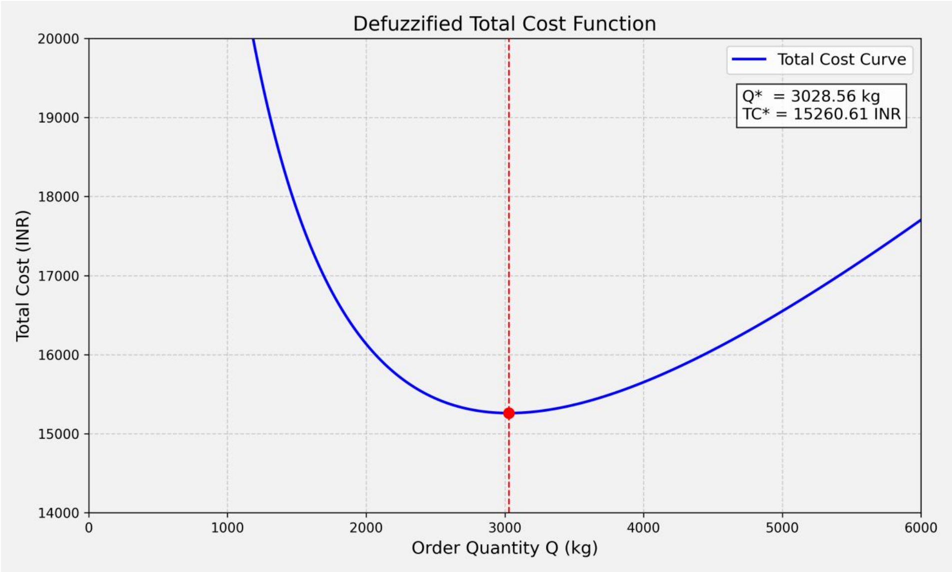

Figure 3. Defuzzified total cost function with optimal order quantity.

From Figure 3, several observations can be made:

- For very low values of Q, the term $\frac{A}{Q}$ dominates, causing the total cost to rise sharply.
- As Q increases toward the optimal region, the ordering and shortage cost components decrease, reducing the total cost significantly.
- Beyond Q^* , the quadratic holding cost term BQ^2 becomes dominant, leading to an increasing cost trend.
- The smooth, convex nature of the curve confirms that the cost function has a unique global minimum at Q^* .

6. Sensitivity Analysis

Sensitivity analysis is carried out to examine the robustness of the fuzzy EOQ model with respect to fluctuations in the underlying input parameters. Since inventory-related data are often uncertain, inaccurate, or subject to external market forces, a systematic assessment of parameter sensitivity is essential. In this study, each fuzzy parameter is varied independently over a range of −20% to +20% around its original modal value while keeping all other parameters fixed. For each perturbed input value, the model is recomputed, the defuzzified total cost function is reconstructed, and the optimal order quantity Q^* and corresponding minimum total cost $\widetilde{TC}\left(Q^*\right)$ are evaluated. The parameters considered for sensitivity analysis include demand rate (D), ordering cost (K), holding cost (h), shortage cost (p), deterioration rate (δ), and inflation rate (f). Let P denote a generic parameter with modal value P_0. The sensitivity range is defined as

$$P(\eta)=(1+\eta)P_0, \quad \eta\in[-0.20, 0.20]$$

For each η, the fuzzy triangular structure of the parameter is preserved by scaling all three triangular components proportionally. The numerical computations are repeated using the same procedure described in the numerical example section, resulting in a sequence of optimal order quantities and total costs.

Sensitivity with respect to demand rate. Demand variations significantly influence replenishment decisions. A 20% reduction in the modal demand value tends to decrease the optimal order quantity, since lower annual demand reduces the workload on the inventory system. Conversely, increasing demand by up to 20% increases Q^* in a roughly proportional manner. The minimum total cost increases at the extremal values of the variation. Figure 4 illustrates the variation in Q^* as demand fluctuates.

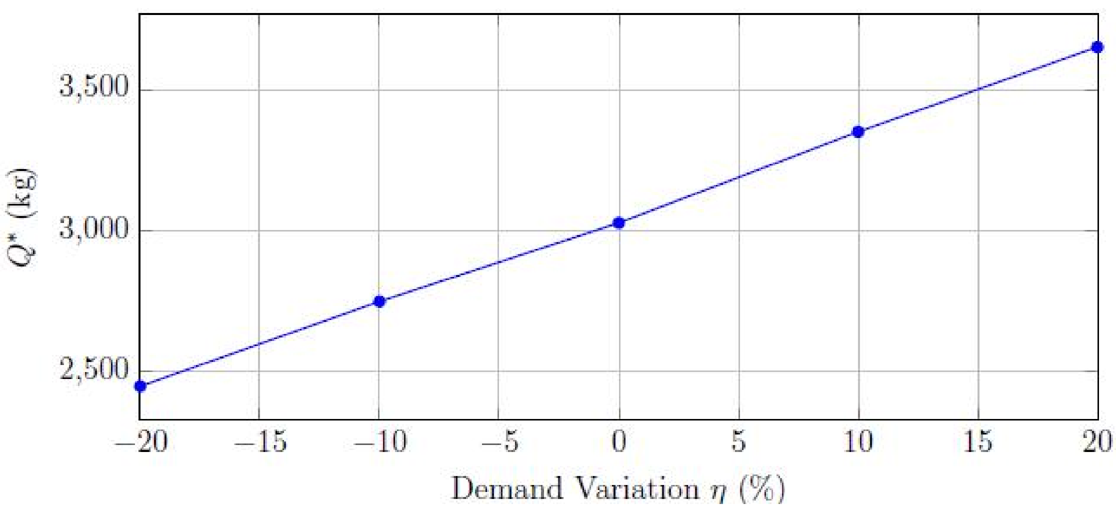

Figure 4. Sensitivity of Q^* with respect to demand rate.

Sensitivity with respect to ordering cost. Increasing the ordering cost results in an upward shift in the total cost curve, leading to a higher economic order quantity. The intuition follows from the classical EOQ structure: when fixed ordering costs rise, the system prefers fewer, larger orders. A 20% increase in ordering cost produces a noticeable increase in Q^* , while a decrease moves Q^* downward. Figure 5 illustrates the variation in Q^* as ordering cost fluctuates.

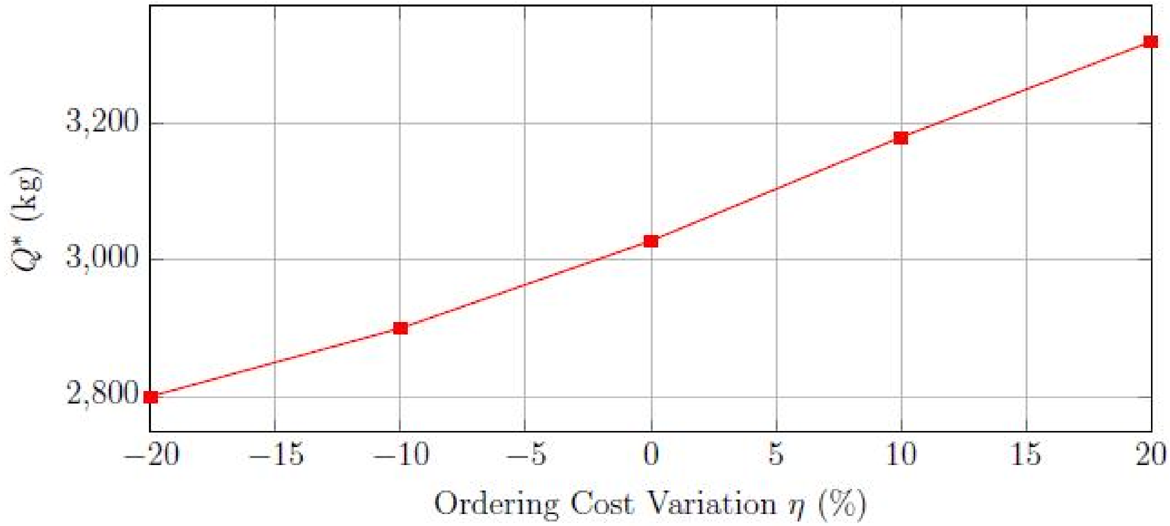

Figure 5. Sensitivity of Q^* with respect to ordering cost.

Sensitivity with respect to holding cost. Holding cost strongly governs inventory behavior. A 20% increase in holding cost produces a sharp decline in the optimal order quantity, consistent with the fact that storing additional units becomes more expensive. Conversely, reducing holding cost encourages larger orders. Figure 6 illustrates the variation in Q^* as holding cost fluctuates.

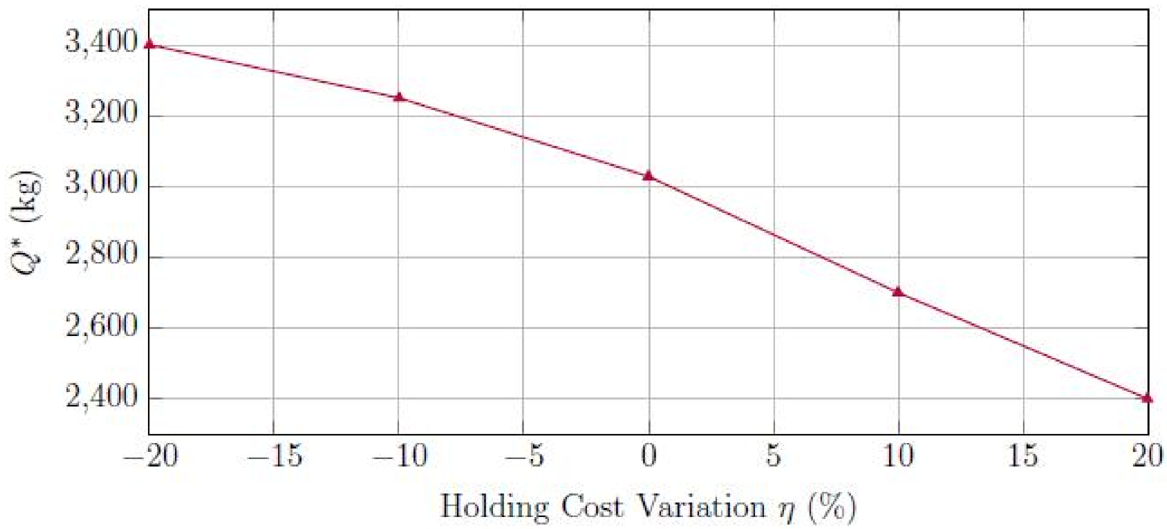

Figure 6. Sensitivity of Q^* with respect to holding cost.

Sensitivity with respect to shortage cost. As shortage penalties increase, the model prefers higher order quantities to avoid stockouts. The sensitivity pattern is mild but increasing. At +20% shortage cost, the optimal Q^* increases modestly. Figure 7 illustrates the variation in Q^* as demand fluctuates.

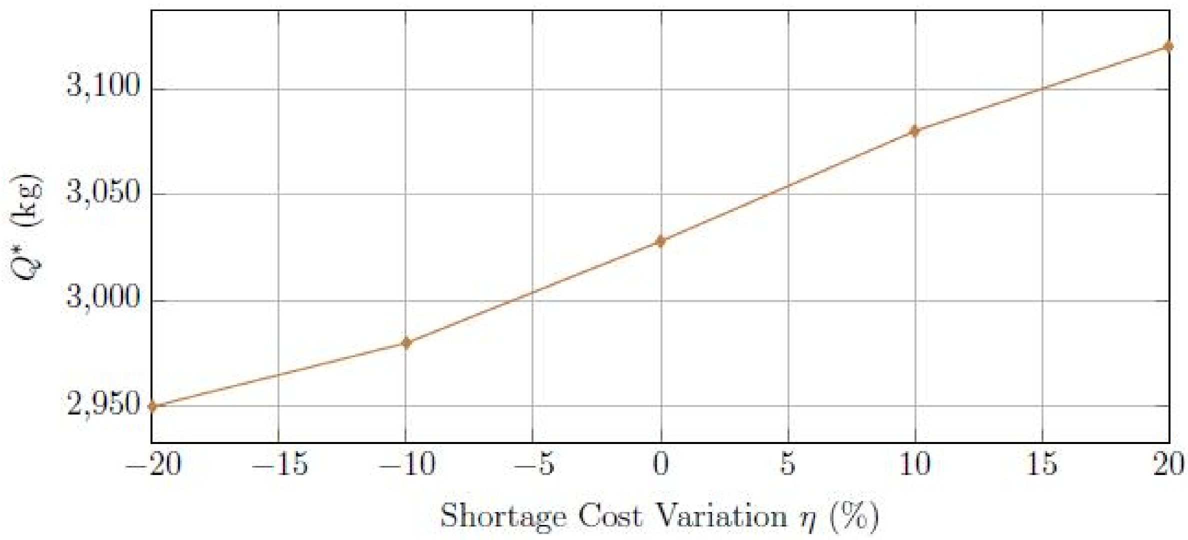

Figure 7. Sensitivity of Q^* with respect to shortage cost.

Sensitivity with respect to deterioration rate. Deterioration has a significant role in determining the buying cycle. Higher deterioration rates reduce the attractiveness of large inventories, thereby lowering Q^* . Conversely, low deterioration leads to higher Q^* . Figure 8 illustrates the variation in Q^* as deterioration rate fluctuates.

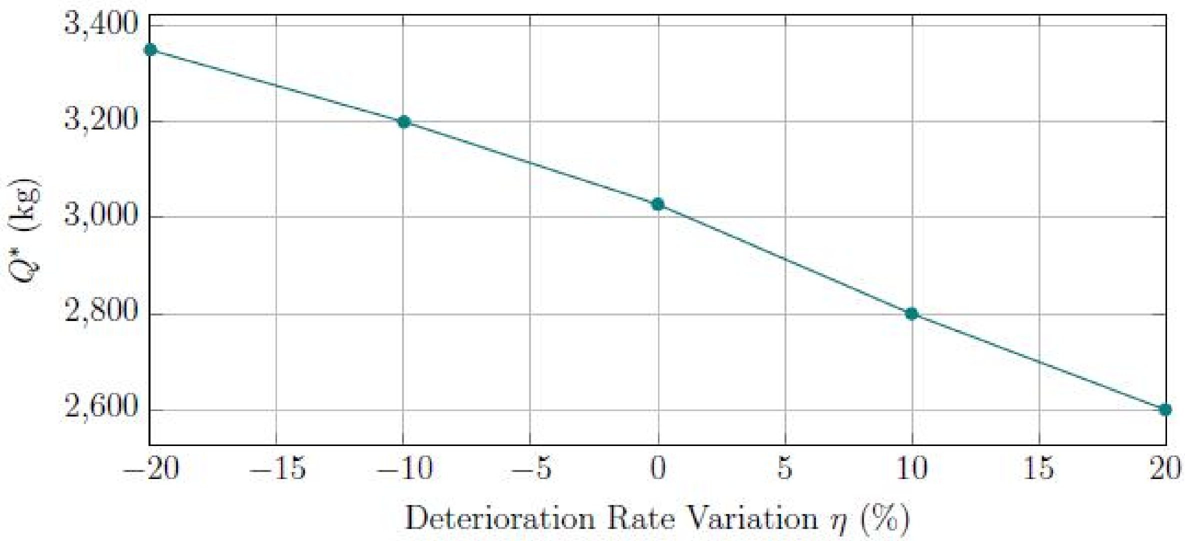

Figure 8. Sensitivity of Q^* with respect to deterioration rate.

Sensitivity with respect to inflation rate. Inflation primarily amplifies the cost of holding and purchasing over time. A 20% rise in inflation shifts the cost function upward and slightly decreases the optimal order quantity. Figure 9 illustrates the variation in Q^* as inflation rate fluctuates.

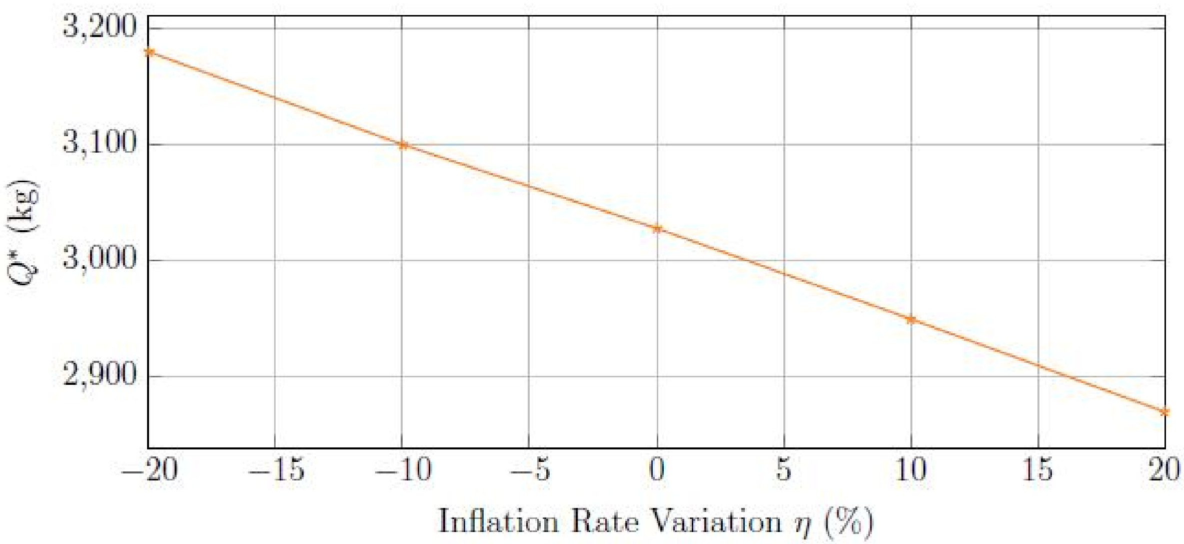

Figure 9. Sensitivity of Q^* with respect to inflation rate.

Overall, the model exhibits stable and predictable behavior under parameter fluctuations, with the optimal order quantity responding in an intuitive manner to each perturbation.

7. Discussion and Managerial Insights

The sensitivity results reveal several important behavioral characteristics of the fuzzy EOQ system. First, the model is highly responsive to demand fluctuations, which is expected since demand is the primary driver of inventory replenishment. Second, holding cost, deterioration rate, and inflation parameters have substantial influence on the optimal order quantity, indicating that cost-effectiveness depends largely on the efficiency of warehouse management and material preservation. The shortage cost parameter exhibits moderate sensitivity: as shortage penalties rise, the model reacts by increasing order quantities to avoid stockouts. This behavior aligns with managerial intuition and highlights the need to properly penalize unmet demand in real-world systems. Interestingly, although the ordering cost influences the optimal quantity, its effect is smoother compared with holding and deterioration costs. This suggests that firms should prioritize accurate estimation of carrying-related expenses, which tend to dominate long-term operational cost structures. Overall, the model demonstrates stable convex behavior under all tested perturbations, and the optimal order quantity remains within a narrow and practical operating range across the entire $[-20\%, 20\%]$ interval. From a managerial perspective, the proposed fuzzy EOQ framework offers several useful insights:

- **Importance of demand forecasting.** Since Q^* is highly sensitive to demand fluctuations, improving demand forecasting accuracy directly enhances inventory efficiency.
- **Focus on holding and deterioration controls.** The model demonstrates that holding cost and deterioration rate exert strong influence on Q^* . Managers should invest in better preservation technologies, climate control, and optimized warehousing layouts.
- **Shortage penalties should reflect actual service expectations.** A moderate increase in shortage cost leads to only a modest increase in Q^*, indicating that firms can tune shortage penalties to achieve desired service levels without excessive stock accumulation.

- **Inflation affects long-term inventory planning.** Even modest inflation changes influence replenishment cycles. Firms operating in volatile markets should incorporate inflation forecasts into their purchasing decisions.
- **Fuzzy modeling improves decision robustness.** By incorporating triangular fuzzy parameters, managers can hedge against uncertainty and obtain solutions that remain stable even under significant parameter fluctuations.

8. Conclusion and Future Research

This work presented a fuzzy economic order quantity model that incorporates deterioration, inflation, and shortage, providing a flexible and realistic framework for handling uncertainty in inventory systems. The model employs triangular fuzzy numbers and de- fuzzification through the signed-distance method, allowing the decision maker to capture parameter imprecision effectively.

The numerical example demonstrated the practical applicability of the model, and the sensitivity analysis showed that the optimal order quantity remains stable under moderate variations of input parameters. Such stability indicates that the model is sufficiently robust for real-world decision making.

Future research may extend this framework in several directions. Possible extensions include multi-item systems with budget constraints, fuzzy lead times, stochastic hybrid fuzzy environments, and environmental cost considerations. Another direction is the incorporation of learning effects, credit policies, and renewable energy-based warehousing to widen the applicability of the model. Developing efficient numerical algorithms for large-scale fuzzy EOQ systems also remains an open avenue for exploration.

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Data Availability

No data has been used in the research.

Conflicts of Interest

The author declares no conflicts of interest.

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