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Research Articles

Comparative Modal Analysis of Metallic and Composite Beam Configurations for Aerospace Structural Applications

Muhammad Mubashir[1*], Sarfraz Shoukat[2], Anas Asim[3], Ahmed Kadhim Zarzoor[4], Rayyan Qaisar[5], Naji Ullah[6], Abdul Ghaffar[6], Muhammad Shoaib-Ur-Rehman[1]

[1]*University of Engineering and Technology, Lahore 54890, Pakistan*

[2]*Trine university, 1881 campus commons dr, Reston, VA 20191, USA*

[3]*National Textile University, Faisalabad, Pakistan*

[4]*University of Al Qadisiyah, 58001 Al-Qadisiyah, Iraq*

[5]*University of Engineering and Technology (UET), Taxila, Pakistan*

[6]*The University of Adelaide, SA 5005, Australia*

KEYWORDS

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modal analysis
aerospace spars
graphite-epoxy composites
T-beam vibration

ABSTRACT

The study of dynamic response of structural members is of great importance in aerospace design where the structural members such as wing spars should not resonate for ensuring the flight safety and structural integrity. Although a lot of research has been conducted on vibration of beams, a comprehensive comparison between metallic and composite cantilever beam and its application to aerospace-related cross-sections has so far not been done. The proposed study is focused on analytical formulations, Rayleigh-Ritz variational approach and finite element analysis (FEA) to systematically evaluate and compare modal characteristics of Aluminum and graphite-epoxy cantilever beam with three geometries T-shape, I-shape and rectangular shape. The closed-form Euler-Bernoulli beam equations and Rayleigh-Ritz approximations was implemented with the help of MATLAB and ANSYS mode-shape contours for the first three bending modes were used for visual validation of the numerical work. The results show very good consistency in the determination of the first bending mode among three implemented methods but for higher order modes variational method show the deviation of 12% in frequency magnitude as compared to analytical and FEA techniques. Among the three tested configurations (I-shape, Rectangular and T-shape), I-shape beam shows higher frequency because of its greater bending stiffness (I/A) as compared to other shapes. Material comparison further highlighted that composite beam shows higher frequency as compare to metallic because of its higher specific modulus (E/ρ). Overall, even some minor variations were observed among the results for three methods, but still all three approaches predicted the same trends for frequency, which proves that the applied methodology and MATLAB codes developed for current research work can be implemented as an application of frequency estimation for aerospace structural components.

*CORRESPONDING AUTHOR:
Muhammad Mubashir; Mechanical Engineering Department, University of Engineering and Technology, Lahore 54890, Pakistan; Email: 2019msmed09@student.uet.edu.pk

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1. Introduction

The compelling nature of aerospace structures and the requirements of their operation and performance make them susceptible to the dynamic loads such as aerodynamic forces, engine vibrations, and turbulence. A detailed case of such a problem is the modal behavior of aircraft wings, that is one of the major components of the aircraft which carry the load, create the lift and ensure structural integrity under various loading conditions. Spars are the main structural element found in the wing that give the wing stiffness and strength to withstand it against bending and torsional loading during flight conditions. These spars are usually available in various shapes such as I-section, T-sections, C-section, rectangular cross-sections etc. and can be approximated as a cantilever beam due to one end fix condition of wing with fuselage, as show in Figure 1. The natural frequency of structural element (spar) may be excited under the previous mention dynamic loads which has the risk of resonance, a situation where external frequency act in harmony with the natural frequency of the structure and cause destruction [1]. Unless carefully examined and counteracted, resonance may cause disastrous structural failures that can put the functionality and safety of the vehicle at risk. Furthermore, the resonance can give rise to critical consequences such as fatigue of material, permanent structural deformation and, failure of load bearing elements [2,3]. Hence, to prevent the resonance and related failures for spars estimation of their natural frequencies by implementing modal analysis method play an important role for safety of aerospace applications. Therefore, when designing aerospace beam structures, the design should be safe and efficient by precise estimation of natural frequencies and mode shapes.

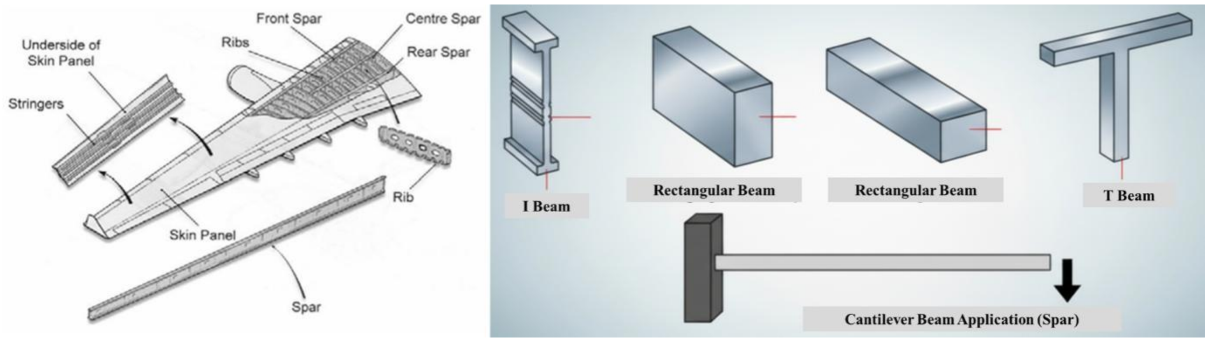

Figure 1. Basic structure of aircraft wing and spars configurations as cantilever beam.

Determination of modal characteristics of beam structures have received a lot of research attention in literature [1-3] because of its criticality of predicting resonance induced failures that must be avoided in both aerospace and mechanical systems. Jiang et al. [4] have suggested an improved vibration model of straight beam, which is inflated, that utilizes the beam theory of Timoshenko by incorporating the tip effects, as well as the addition of the mass of the gas, in refining the accuracy of natural frequencies prediction. The work focused on structural geometry and internal gas that have a great impact on dynamic response and gave information that can be relevant to lightweight aerospace structures (inflatable boom and deployable wings). Walunj et al. [5] conducted modal testing of rectangular cantilever beam in the format of impact hammer testing and LabVIEW. They indicated a good correlation in experimental and theoretical frequencies where it was revealed that aluminum beams had higher natural frequencies and damping than mild steel. Mekalke et al. [6] conducted their research for estimation of frequency for the cantilever beams fabricated from different materials and geometric dimensions. They compare analytical, experimental and ANSYS results and found they all are in good agreement with each. Their results revealed that the natural frequencies have a negative dependence on material density and resonance can be avoided by accurately predicting vibration modes a principle essential to aerospace component design. In the studies of Mia et al. [7] and Arun Kumar et al. [8], the vibration behavior of cracked cantilever beams was examine using ANSYS and validated against experimental studies. The two studies affirmed that the presence of cracks decreases the natural frequency and the extent of the decrease is also dependent on the location and depth of the cracks. The results play a key role in forecasting fatigue induced failure in aerospace structures, especially in the parts of the structure such as wing spars and landing gear beams that experiences cyclic loading. Talekar et al. [9] used the first-order theory of shear deformation to study layered composite cantilever beam, showing the influence of the lay-up sequence, fiber orientation and length to thickness ratio on the modal parameters. From this aspect, Sujith et al. [10] investigated smart active composite beams reinforced with shape memory alloys (SMA) where close correlation of analytical and FEA findings was established. Based on their research they recommended that beams reinforce with SMA have the potential to be used in the vibration control of adaptive aerospace structures. Finite element modal analysis of the cantilever beam fabricated from two different natural fiber composites (nettle/polyester and chicken feather/epoxy) were done by Pankaj et al. [11], in their research they prepared various samples for two materials and found their frequency ranges. Based on their research they identified nettle/polyester as more sustainable material for beam structure of cantilever type. Turkay et al. [12] fabricated hybrid wood-steel composite beams by using different layering combination of steel and wood for achievement of varying stiffnesses of each fabricated beam. After fabrication they performed operational modal analysis on them and estimate their frequencies. They demonstrated that hybrid beam shows better modal characteristics as compared to the literature, and their beam can be used for the hybrid aerospace structure applications. An investigation on modal properties of laminated carbon-ceramic beams that act as analogy of turbine blades was conducted by Faiza et al. [13] indicated that composite laminates have excellent natural frequencies and stiffness as compared to superalloys. Their results highlighted that frequency optimization can help in increasing turbine blades safety and efficiency against resonance. Ramesh et al. [14] also made another important contribution, investigating the dynamic behavior of mixed carbon-fiber composites at different strain rates in ANSYS and revealing eight vibration modes and directly describing the change in frequency based on the material arrangement. Ahiwale et al. [15] examined cracked cantilever beam of mild steel for different locations, based on their research they established that presence of cracks at the upper or bottom surface of the beam decrease its natural frequency much more than the crack present in central surface. The authors based on their conclusions highlight the significance of structural health monitoring with modal behavior analysis.

Based on above literature review it can be seen that in above studies, much effort has been applied on modal analysis of metallic, composite, and cracked beams, by using the analytical, numerical, and experimental methods. However, there is no study have been conducted yet in which the investigation of the modal behavior of metallic and composite cantilever beam having cross-sections (I, T and rectangular shape) under the same boundary conditions, by using three different methods such as conventional (analytical), variational (Rayleigh-Ritz), and Finite element (ANSYS) methods was analyzed. Hence, this research is an effort to fill up this gap by doing the analysis of three cantilever beams by using three methods and their comparison with each other. The results obtained based on this research will help in determining the suitability and application of particular beams for aerospace structures.

2. Materials and Methods

The present research uses three complementary methods: analytical formulation, Rayleigh Ritz method, and the Finite Element Analysis (FEA) to identify and compare the natural frequency of three beam designs (I-beam, T -beam and rectangular beam) made of metallic and composite materials. The detail of each modeling is discussed in subsequent sections.

2.1. Analytical Formulation

The governing differential equation for uniform cantilever beam is presented in eq (1) [1-3],

$$EI\frac{d^4w(x)}{dx^4} = \rho A\frac{d^2w(x)}{dt^2} \tag{1}$$

The essential boundary conditions of cantilever beam presented in eq (2),

$$w(0) = 0, \quad w^{'(0)} = 0, \quad w^{''(L)} = 0, \quad w^{'''(L)} = 0 \tag{2}$$

Solving eq (1), with essential boundary conditions of cantilever beam yields eq (3),

$$\cosh(\beta_n L)\cos(\beta_n L) + 1 = 0 \tag{3}$$

The solution of above equations gives the expression of natural frequencies presented in eq (4),

$$\omega_n = \beta_n^2 \sqrt{\frac{EI}{\rho A L^4}}, \quad f_n = \frac{\omega_n}{2\pi} \tag{4}$$

2.2. Rayleigh–Ritz Method Formulation

Rayleigh-Ritz method, is a variational technique that uses for approximation of vibration modes by assuming a trial function that is denoted by $\phi_i(x)$. The trial function is assumed to meet geometric boundary conditions, with the deflection being expressed as follows [3, 19, 20]:

$$w(x,t) = \sum_{i=1}^{N} a_i\,(t)\phi_i(x) \tag{5}$$

Substitute into the total energy:

$$T = \frac{1}{2}\int_0^L \rho\, A\left(\frac{\partial w}{\partial t}\right)^2 dx, \quad U = \frac{1}{2}\int_0^L E\, I\left(\frac{\partial^2 w}{\partial x^2}\right)^2 dx \tag{6}$$

Then, apply Lagrange's equations:

$$\frac{d}{dt}\left(\frac{\partial (T-U)}{\partial \dot{a}_i}\right) - \frac{\partial (T-U)}{\partial a_i} = 0 \tag{7}$$

Leading to the eigenvalue problem:

$$[K]\mathbf{a} = \omega^2 [M]\mathbf{a} \tag{8}$$

$$K_{ij} = \int_0^L E\, I\, \phi_i^{''(x)\phi_j{''}(x)}\, dx, \quad M_{ij} = \int_0^L \rho\, A\, \phi_i(x)\phi_j(x)\, dx \tag{9}$$

2.3. Beam Geometry, Dimensions, and Material Properties

Three different beam configurations (rectangular, T-section, and I-section) were created using equal bounding dimensions for present studied so their modal characteristics could be reliably compared with each other. All three beams had a total length of 8.8 m, which is a typical span for aerospace structural members like wing spars. The rectangular beam with width of 450 mm and height of 136 mm, was selected as a baseline geometry for present study. The T-section beam had a flange of 450 mm wide with a thickness of 16 mm, a total section height of 136 mm, and a web 22 mm thick that extends downward from the flange. The I-section beam used identical outside dimensions but included both a top and bottom flange each 450 mm wide and 16 mm thick, separated by a center web of 22 mm thickness and 104 mm high. The cross section of each beam is presented in Figure 2.

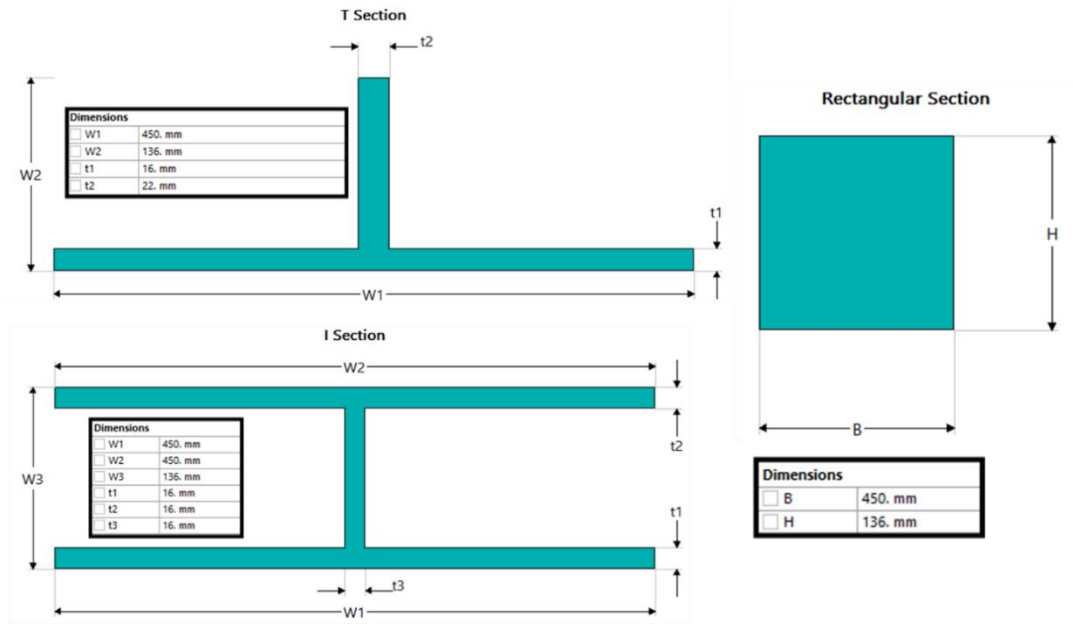

Figure 2. Cross Section dimension of selected beam.

The material properties used in this study were selected from established literature [4,5] to ensure realistic representation of metallic and composite aerospace structural materials. For the metallic configuration, an aluminum alloy commonly used in aircraft structural components was considered. For the composite configuration, AS4/3501-6 graphite/epoxy one of the most widely referenced aerospace-grade unidirectional composites was adopted. The detailed material parameters employed in the analytical, Rayleigh–Ritz, and finite element analyses are summarized in Table 1, covering elastic constants, strength properties, and density for both materials.

Table 1. Material properties utilized for analytical, Rayleigh–Ritz and FE analysis.

Parameter	Symbol	Aluminum Alloy [5]	AS4/3501-6 Composite [4]	Unit
Elastic Modulus (metal) / Longitudinal Modulus (comp.)	E , E_1	75.8	144.80	GPa
Transverse Modulus	E_2	—	9.65	GPa
Shear Modulus	G_{12}, G_{13}	—	4.14	GPa
Shear Modulus	G_{23}	—	3.45	GPa
Poisson's Ratio	ν, ν_{12}	0.30	0.30	–
Yield Strength	σ_y	397	—	MPa
Ultimate Tensile Strength	σ_u	442	—	MPa
Density	ρ	2770	1389.23	kg/m^3

2.4. MATLAB Implementation for Analytical and Rayleigh–Ritz Methods

For computation of frequency by using analytical and Rayleigh-Ritz (RR) methods a MATLAB code specifically for execution of present research was developed. The MATLAB code procedure start by importing the geometry information, including cross-section sizes of rectangular, T-shaped, and I-shaped beams, and the material properties of the aluminum and composite. After completion of all essential inputs for code, MATLAB program automatically computed all the important parameters required for computing natural frequency including cross-sectional area, the second moment of area (I_{yy}) and the mass per unit length evaluated by density. For analytical calculations, the closed-form expressions based on eq (1-4) was used in code for computation of natural frequencies for cantilever beams. For RR method eq (5-9) were incorporated, the script initially assumed an approximate mode shape function, then constructs mass and stiffness matrices with the aid of numerical integration and finally resolves the generalized eigenvalue problem. At the end code computes the natural frequencies of all the modes for given material and boundary condition. The flow chart for this is presented in Figure 3.

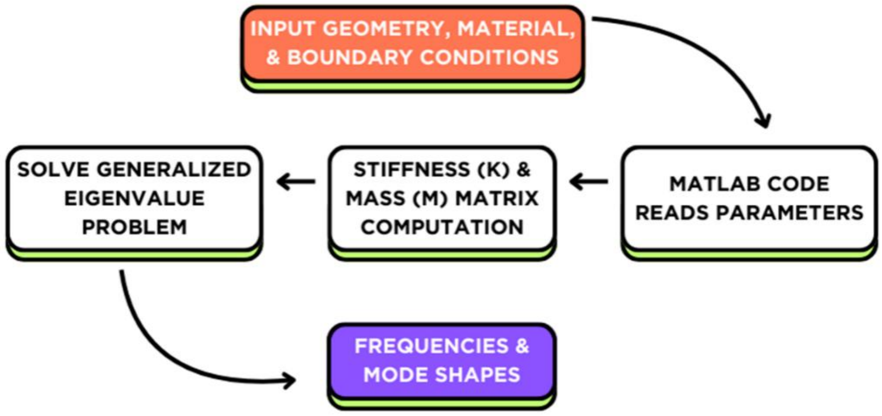

Figure 3. MATLAB implementation workflow for analytical frequencies estimation.

2.5. Finite Element Analysis in ANSYS

For all cantilever beam configurations used in present study, ANSYS Workbench was used to run their finite element analysis for determination of their natural frequencies and later a comparison was made with analytical and Rayleigh Ritz predictions. The geometry of each beam was created by using ANSYS Space claim a CAD module of ANSYS. The geometry of each beam was modeled using line body and specific cross section shown in Figure 1 was assigned to it. The mechanical properties for composite and metal assigned to each beam for conducting analysis are presented in table 1. One end of the beams was assumed as a fixed to mimic the cantilever condition and used as boundary condition for analysis as shown in Figure 4. For meshing beam elements of quadratic order with 100 number of divisions was utilized for each beam as presented in Figure 5. The 100 number of divisions were selected based on mesh convergence by following the guidelines presented in literature [21,23]. After completing the meshing and all relevant inputs 20 mode shapes were requested in analysis setting window of modal analysis for both metal and composite beam with all configurations.

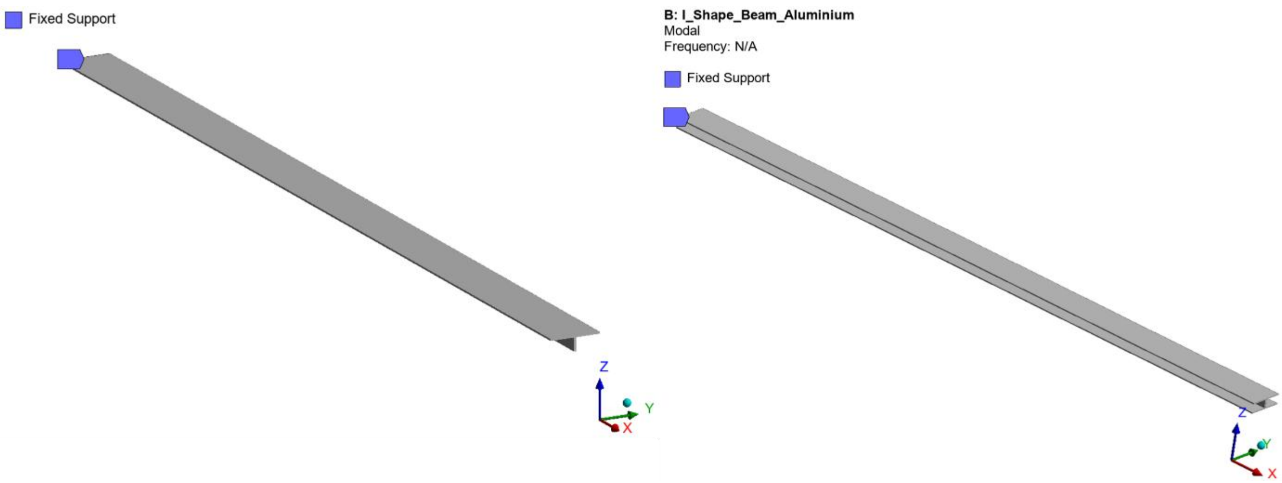

Figure 4. Boundary conditions applied to beam for FE Analysis.

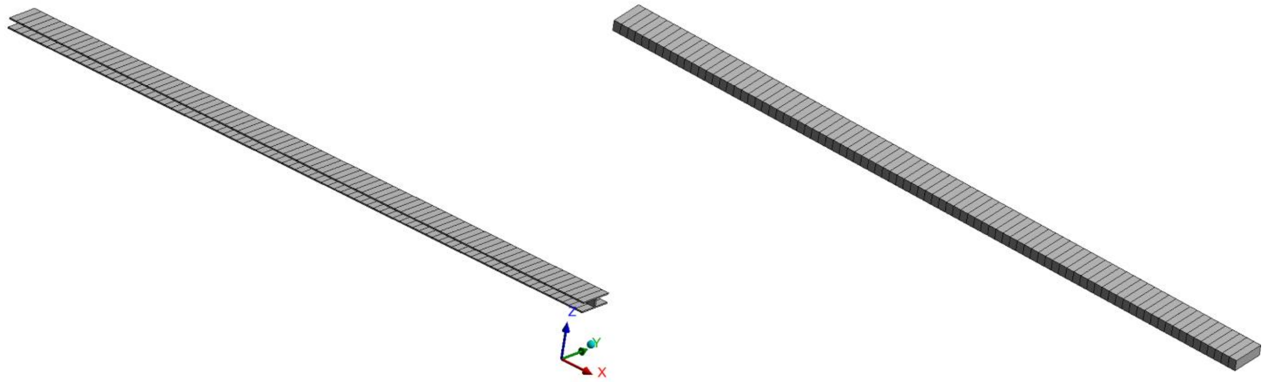

Figure 5. Meshing of beam applied in FE Analysis.

3. Results

The natural frequencies obtained from Analytical, Rayleigh–Ritz, and Finite Element Analysis (FEA) methods for T-, I-, and rectangular beam configurations are summarized in Tables 2–4. The results reveal consistent trends across all three methods, while also highlighting the inherent differences between simplified analytical models, approximate variational solutions, and high-fidelity finite-element predictions.

Table 2. Natural frequencies comparison for T-Shape beam.

Parameter	Material	T-Shape Frequency (Hz)		
		Analytical	Reyleigh-Ritz	FEM
Mode 1	Aluminum	1.334	1.129	1.333
	Graphite Epoxy	2.603	2.204	2.579
Mode 2	Aluminum	8.360	7.076	8.324
	Graphite Epoxy	16.315	13.81	15.331
Mode 3	Aluminum	23.407	19.815	23.169
	Graphite Epoxy	45.683	38.673	39.855

Table 3. Natural frequencies comparison for I-Shape beam.

Parameter	Material	I-Shape Frequency (Hz)		
		Analytical	Reyleigh-Ritz	FEM
Mode 1	Aluminum	2.184	1.882	2.149
	Graphite Epoxy	4.263	3.673	4.041
Mode 2	Aluminum	13.69	11.794	13.259
	Graphite Epoxy	26.718	23.018	20.986
Mode 3	Aluminum	38.332	33.027	36.249
	Graphite Epoxy	74.811	64.457	68.409

Table 4. Natural frequencies comparison for Rectangular-Shape beam.

Parameter	Material	Rectangular Shape Frequency (Hz)		
		Analytical	Reyleigh-Ritz	FEM
Mode 1	Aluminum	1.484	1.309	1.4838
	Graphite Epoxy	2.896	2.555	2.890
Mode 2	Aluminum	9.300	8.206	9.2881
	Graphite Epoxy	18.151	16.015	17.860
Mode 3	Aluminum	26.041	22.979	25.960
	Graphite Epoxy	50.824	44.847	48.947

4. Discussions

This section presents the discussions of results obtained above

4.1. Comparison Across Methods

Based on tabulated results presented in table 2-4, it can be observed that the Analytical, Reyleigh-Ritz (variational method) and FEM frequencies are in good agreement across all beam shapes for both Aluminum and Graphite Epoxy material and the results show a small deviation that falls within 3% for the first bending mode. This good agreement among the results for first mode indicates that the applied analytical formulation, assumed closed-form formulation for variational method and applied FE assumptions are fair accurate for capturing the modal behavior of cantilever beam with specific cross-sectional shape. Analytical and FEM frequencies show good agreement for higher bending modes (2nd and 3rd) as well.

However, for higher mode the Rayleigh-Ritz technique estimates slightly reduced frequencies as compared to analytical and FEM with a difference of 5% for 2nd mode to 12% for 3rd mode. The main reason for this difference is the introduction of an approximation error in initially assumed admissible (global) functions that used for estimation of frequencies in Rayleigh-Ritz technique. The growing complexity of mode shapes cannot be described well by a small base of global functions is another reason for this difference. However, even some minor variations are present among the results, but still all three approaches predicted the same trends for frequency, which proves that the MATLAB codes developed for current research work are able to simulate the modal behavior and frequencies of the beams in an effective way.

4.2. Influence of Cross-Sectional Geometry

Among the three tested configurations (I-shape, Rectangular and T-shape), cross-sectional geometry has a strong impact on the modal performance of beam, which is clearly revealed in the natural frequency trends chart shown in Figure 6. The chart shows the variation of frequency magnitude for first mode among three tested configurations for Aluminum material. The 2nd and 3rd bending modes also reveal same sort of trend. Based on Figure 6, it can be seen that the I-shape beam shows higher frequency among all three methods followed by rectangular and T-shape beam. This is mainly because the I-beam exhibits greater bending stiffness (I/A) that is define as the ratio of moment of inertia (I) to cross sectional area (A). The mathematically frequency can be expressed as the proportional to square root of bending stiffness as follow,

$$f = \sqrt{\frac{EI}{\rho A}} = \left(\sqrt{\frac{E}{\rho}}\right)\left(\sqrt{\frac{I}{A}}\right) = C\sqrt{\frac{I}{A}} \tag{10}$$

$$f \propto \sqrt{\frac{I}{A}} \tag{11}$$

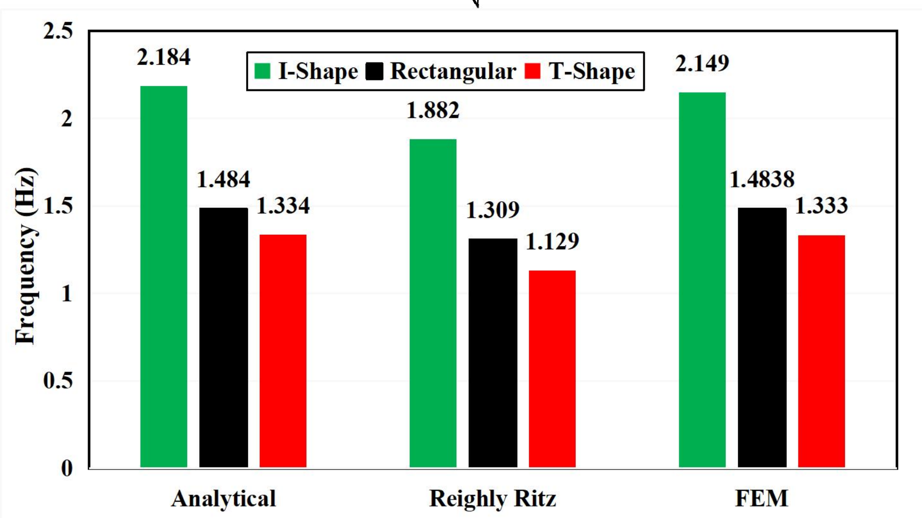

Figure 6. Frequency comparison among different cross-sectional geometries.

Because I-shape beam has high moment of inertia formed by the wide flanges that are uniformly distributed and lower cross-sectional area makes it bending stiffness higher as compared to the other shapes and hence shows higher frequencies. On the other hand, the rectangular and T-shape beam exhibits intermediate and lower frequencies, indicating moderate and lower bending stiffness.

4.3. Influence of Material Type

The results for influence of material type can be described based on term known as specific modulus $\left(\frac{E}{\rho}\right)$ and is given by using eq (12)

$$f \propto \sqrt{\frac{E}{\rho}} \tag{12}$$

Based on eq (12), frequency magnitude highly depends on the specific modulus of a material for two beams having same cross-sectional dimensions. For composite specific modulus (E/ρ), is often 1.8 to 2 times higher than metal counterpart that indicates for same cross-sectional area and dimensions, a beam made of composite material shows higher natural frequency as compared to metal one. This trend can also be verified from Figure 7 as well presented below.

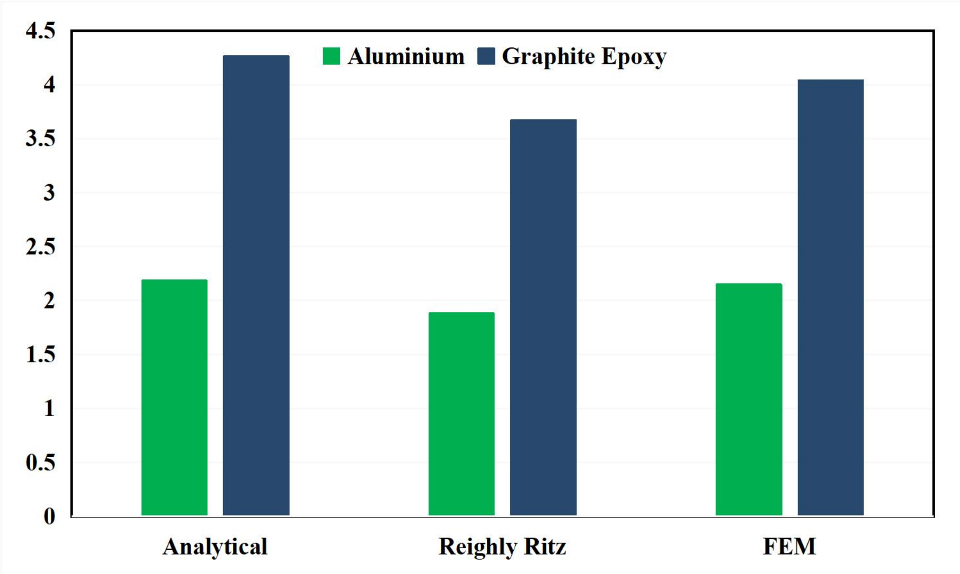

Figure 7. Frequency comparison among aluminum and graphite epoxy.

4.4. ANSYS Contours and Aerospace Applications

ANSYS mode-shape contours for the first three bending modes are presented in Figure 8 to 10 to make visual validation of the mode shapes of the numerical work. These contour plots not only verify the correctness of the FEM formulation but also predicted aerospace-specific structural behavior of the beam. The initial bending mode, which is in Figure 8, has a smooth upward curve that resembles the wing of an aircraft lifting during an aircraft flight. This is the most vital mode in preventing resonance as external excitations (engine vibrations, gust loads) usually coincide with low-frequency range. In the second mode contour in Figure 9, there is only one node (inflection point), at which the curvature is reversed. This trend is equivalent to mid-span structural flexibility and such behavior is directly pertinent to aeroelastic flutter analysis, where wing stability is influenced by higher-order interactions between bending. The third mode of bending shown in Figure 10 has two points of inflexion and it shows a more complex dynamic response.

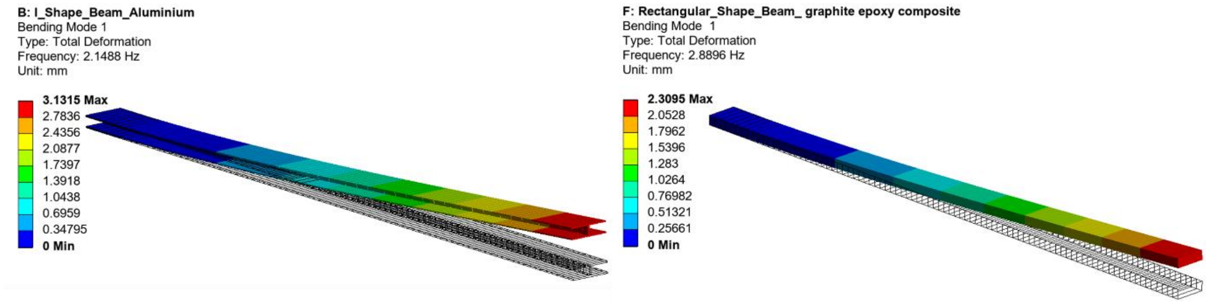

Figure 8. First bending mode.

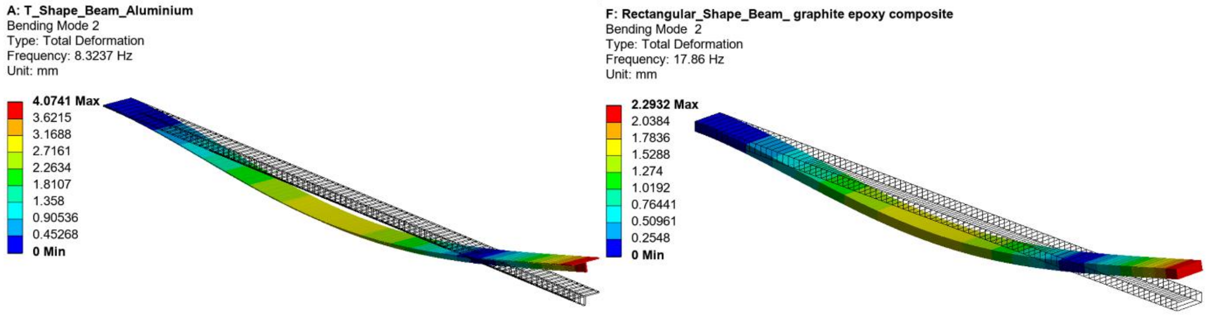

Figure 9. Second bending mode.

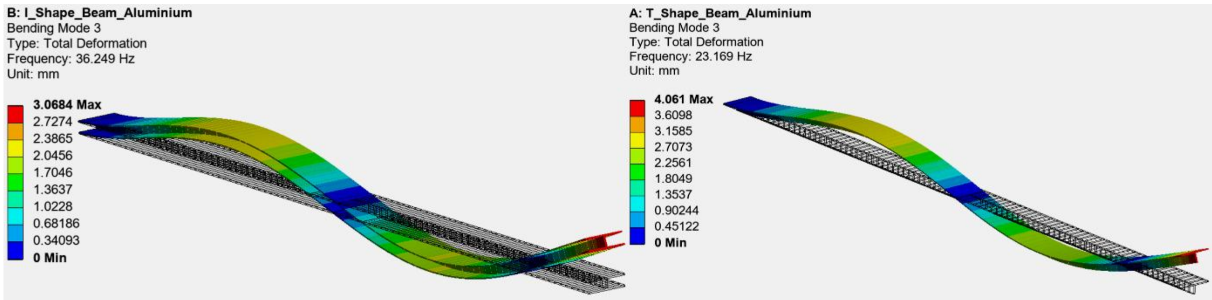

Figure 10. Third bending mode

This mode is significant in high-level aerospace structural health parameters because the higher-order modes are sensitive to local stiffness loss.

5. Conclusions

This study presented a comprehensive modal analysis of three beam configurations (I-shape, Rectangular and T-shape) created using two materials Aluminum and graphite epoxy composite by implementing three different techniques (analytical, variational and FEM). The results obtained by all three methods shows a small deviation that falls within 3% for the first bending mode. However, for higher mode analytical and FEM shows good agreement but variational method shows a difference of range 5 to 12 percent. The main reason for this difference is the introduction of an approximation error in initially assumed admissible (global) functions that used for estimation of frequencies in variational technique. After verifying the effectiveness among applied techniques, the influence of cross-sectional geometry was then analyzed. Among the three tested configurations (I-shape, Rectangular and T-shape), I-shape beam shows higher frequency among all three methods followed by rectangular and T-shape beam. This is mainly because the I-beam exhibits greater bending stiffness (I/A) as compared to other shapes. Material comparison further highlighted that frequency is highly depend upon its specific modulus (E/ρ). For composite specific modulus (E/ρ), is often 1.8 to 2 times higher than metal counterpart that indicates for same cross-sectional area and dimensions, a beam made of composite material shows higher natural frequency as compared to metal one. Furthermore, ANSYS mode-shape contours for the first three bending modes are also presented to make visual validation of the mode shapes of the numerical work. The initial bending mode, has a smooth upward curve that resembles the wing of an aircraft lifting during an aircraft flight. This is important mode for preventing resonance as external excitations usually coincide with low-frequency range. In the second mode, there is only one inflection point, at which the curvature is reversed. This trend is equivalent and applicable to aeroelastic flutter analysis, where wing stability is influenced by higher-order bending. The third mode of bending has two points of inflexion and it shows a more complex dynamic response. However, even some minor variations were observed among the results for three applied methods, but still all three approaches predicted the same trends for frequency, which proves that the applied methodology and MATLAB codes developed for current research work are able to simulate the modal behavior and frequencies of the beams in an effective way and can be used as an application of frequency estimation in aerospace structural components.

Author’s Contributions

Conceptualization, M Mubashir. and Anas Asim.; methodology, Naji Ullah; software, Rayyan Qaisar.; validation, S Shoukat, Abdul Ghaffar. and Z.Z.; formal analysis, M Mubashir.; investigation, Naji Ullah; resources, Abdul Ghaffar.; data curation, M Mubashir.; writing—original draft preparation, Anas Asim and Ahmed Kadhim Zarzoor; writing—review and editing, S Shoukat.; visualization, Ahmed Kadhim Zarzoor; supervision, M Mubashir; project administration, Muhammad Shoaib-Ur-Rehman. All authors have read and agreed to the published version of the manuscript.” Authorship must be limited to those who have contributed substantially to the work reported.

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Data Availability

The data supporting the findings of this published article are provided to journal as a supporting data and can be requested from author as well.

Conflicts of Interest

The authors declare no conflict of interest.

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